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MATDIP301

Third Semester B.E. Degree Examination, December 2012

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the modulus and amplitude of the complex number $1-\cos\alpha + i\sin\alpha$. **(05 Marks)**

b. If z_1 and z_2 are two complex numbers, show that $|z_1+z_2|^2 + |z_1-z_2|^2 = 2\{|z_1|^2+|z_2|^2\}$. **(05 Marks)**

c. Find the fourth roots of $-1+i\sqrt{3}$. **(05 Marks)**

d. If $2\cos\theta = x+\frac{1}{x}$, prove that $2\cos r\theta = x^r + \frac{1}{x^r}$. **(05 Marks)**

2 a. Find the n^{th} derivative of $e^{2x}\cos^3 x$. **(07 Marks)**

b. Find the n^{th} derivative of $\frac{x}{x^2-5x+6}$. **(06 Marks)**

c. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. **(07 Marks)**

3 a. Find the angle between the pair of curves $r = 6\cos\theta$, $r = 2(1+\cos\theta)$. **(07 Marks)**

b. Find the pedal equation of the curve $r^2 = a^2\sin 2\theta$. **(06 Marks)**

c. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$. **(07 Marks)**

4 a. If $u = x^2y + y^2z + z^2x$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = (x+y+z)^2$. **(05 Marks)**

b. If $u = \tan^{-1}\left(\frac{x^3y^3}{x^3+y^3}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \frac{3}{2}\sin 2u$. **(05 Marks)**

c. If $u = x+y+z$, $v = y+z$, $z = uvw$, find Jacobian of x, y, z with respect to u, v, w. **(05 Marks)**

d. If $z = f(x, y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. **(05 Marks)**

5 a. Obtain the reduction formula for $\int_0^{\pi/2}\cos^n x\,dx$ and hence evaluate $\int_0^{\pi/2}\cos^6 x\,dx$ and $\int_0^{\pi/2}\cos^9 x\,dx$. **(07 Marks)**

b. Evaluate $\int_0^1\int_{x^2}^{\sqrt{x}} xy(x+y)\,dy\,dx$. **(06 Marks)**

c. Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$. **(07 Marks)**

6 a. Define Gamma and Beta functions. Show that $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$. **(07 Marks)**

b. Prove that $\int_0^\infty x^2 e^{-x^4} dx \times \int_0^\infty e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$. **(07 Marks)**

c. Evaluate $\int_0^1 (\log x)^6 dx$. **(06 Marks)**

7 a. Solve the equation $\frac{dy}{dx} + x\tan(y-x) = 1$. **(06 Marks)**

b. Solve $x^2 y dx - (x^3 + y^3) dy = 0$. **(07 Marks)**

c. Solve $(e^y + y\cos xy)dx + (xe^y + x\cos xy)dy = 0$. **(07 Marks)**

8 a. Solve the equation $(D^3 + 1)y = 0$, where $D = \frac{d}{dx}$. **(06 Marks)**

b. Solve the equation $(D^2 - 2D + 1)y = xe^x$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$. **(07 Marks)**

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